

Name: Solutions

1. [3 points] Solve the IVP $4y'' + 12y' + 9y = 0$ with $y(0) = 0$ and $y'(0) = 1$.

$$4r^2 + 12r + 9 = 0$$

$$(2r+3)^2 = 0$$

$$r = -\frac{3}{2}, \text{ multiplicity 2.}$$

$$y = c_1 e^{-\frac{3}{2}t} + c_2 t e^{-\frac{3}{2}t}$$

$$y' = -\frac{3}{2}c_1 e^{-\frac{3}{2}t} + c_2 e^{-\frac{3}{2}t} - \frac{3}{2}c_2 t e^{-\frac{3}{2}t}$$

$$y(0) = 0: [0 = c_1 \cancel{e^0}] \text{ ok!}$$

$$\underline{y'(0) = 1: } \underline{1 = -\frac{3}{2}c_1 + c_2 = 0}$$

$$\cancel{c_1 = 0} \quad 2 = -\frac{3}{2}c_1 + c_2$$

2. [3 points] Find the general solution to $y^{(4)} + 8y^{(3)} + 17y^{(2)} = 0$.

$$r^4 + 8r^3 + 17r^2 = 0$$

$$r^2(r^2 + 8r + 17) = 0$$

$$r = 0, \left\{ r = \frac{-8 \pm \sqrt{64 - 4 \cdot 17}}{2} = -4 \pm \sqrt{16 - 17} \right.$$

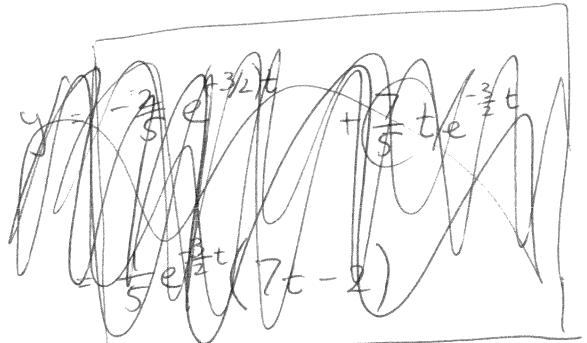
$= -4 \pm i$

$$\begin{cases} y_1 = e^{0t} = 1 \\ y_2 = te^{0t} = t \end{cases} \quad y = e^{(-4+i)t} = e^{-4t} \cdot e^{it} \\ = e^{-4t} (\cos(t) + i \sin(t)) \\ y_3 = e^{-4t} \cos(t) \quad y_4 = e^{-4t} \sin(t)$$

$$\text{So } y = c_1 y_1 + c_2 y_2 + c_3 y_3 + c_4 y_4$$

$$y = \boxed{c_1 + c_2 t + c_3 e^{-4t} \cos(t) + c_4 e^{-4t} \sin(t)}$$

$$C_2 = 2 + \frac{3}{2} = \frac{7}{2}$$



$$\boxed{y = e^{-\frac{3}{2}t} + \frac{7}{2}t e^{-\frac{3}{2}t}}$$

3. [3 points] Find the general solution to $y'' + 2y' + 5y = \sin t$.

(1) Gen Soln to $y'' + 2y' + 5y = 0$:

$$r^2 + 2r + 5 = 0$$

$$r = \frac{-2 \pm \sqrt{4 - 4 \cdot 5}}{2} = -1 \pm \sqrt{1 - 5}$$

$$= -1 \pm 2i$$

$$y = c_1 e^{-t} \cos(2t) + c_2 e^{-t} \sin(2t)$$

$$-A \sin t - B \cos t$$

$$+ 2[-B \sin t + A \cos t]$$

$$+ 5[A \sin t + B \cos t]$$

$$(4A - 2B) \sin t + (4B + 2A) \cos t = \sin t$$

$$[4A - 2B = 1] \cdot 2$$

$$2A + 4B = 0$$

$$10A = 2$$

$$A = \frac{1}{5}, \quad 2B = 4A - 1 = \frac{4}{5} - 1 = -\frac{1}{5}$$

$$B = -\frac{1}{10}$$

$$Y = \frac{1}{5} \sin(t) - \frac{1}{10} \cos(t)$$

Gen Soln:

$$\boxed{y = c_1 e^{-t} \cos(2t) + c_2 e^{-t} \sin(2t) + \frac{1}{5} \sin(t) - \frac{1}{10} \cos(t)}$$

4. [1 point] Given that y_1 is a solution to $y'' + p(t)y' + q(t)y = 0$, the reduction of order procedure looks for additional solutions of the form $y = \underline{\hspace{1cm} v y_1 \hspace{1cm}}$, where

$v(t)$ is a function of t .