Directions:

- This is a take home midterm exam.
- Solve 5 of the following 6 problems.
- All solutions require justification. For computational questions, show your work.
- You are allowed to use the course textbook, notes from class, class homeworks and homework solutions, and non-programmable calculators.
- No other aids are permitted. In particular, no computer/sage assistance is allowed.
- **You may not discuss these problems with anyone except for me** until the exam is past due.
- When submitting your midterm exam, sign the following honor pledge, and include this page.

Honor Pledge

I have not discussed these problems with anyone, except perhaps the instructor. I have not used any unauthorized materials. My work on this exam is my own.

Signed: ____

1. [NT 2-4.2] Without using the fundamental theorem of algebra (i.e. the prime factorization theorem), show directly that every positive integer is uniquely representable as the product of a non-negative power of two (possibly 2^0) and an odd integer.
2. Find all integral solutions (x, y) to the equation $24x + 14y = 8$.
3. Prove or disprove the following. Let $a_1, \ldots, a_r$ be positive even integers, and let $b_1, \ldots, b_s$ be positive integers. If $r \geq s+3$ and $a_i > b_j$ for all i and j, then the quotient $(a_1 a_2 \cdots a_r)/(b_1 b_2 \cdots b_s)$ is either an even integer or a non-integral rational.
4. Find all integers x that satisfy the following system of congruences:
$$9x \equiv 1 \pmod{20} \qquad 52x \equiv 2 \pmod{209} \qquad 8x \equiv 3 \pmod{21}$$
5. Prove that if n is divisible by 11 and n' is obtained from n by inserting two identical digits between consecutive digits of n, then n' is also divisible by 11. For example, since 407 is divisible by 11, the following are also divisible by 11: 22407, 43307, 40997, and 40722.
6. [NT 6-2.5] Prove that $\sigma(n) \equiv d(m) \pmod{2}$ where m is the largest odd factor of n.