Name: ______________________________

Directions: Show all work. Answers without work generally do not earn points. This test has 60 points but is scored out of 50 (higher scores capped at 50).

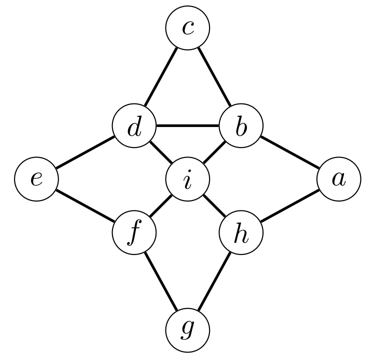
1. **[3 parts, 4 points each]** Recall the n-dimensional hypercube Q_n is the graph whose vertices are the set of all bitstrings of length n where $x_1 \dots x_n$ and $y_1 \dots y_n$ are adjacent if and only if they differ in exactly one coordinate. (For example, 0110 and 1110 are adjacent in Q_4, but 0110 and 1111 are not.)

   (a) Draw Q_2 and Q_3.

   (b) Give a formulas for the number of vertices and the number of edges in Q_n.

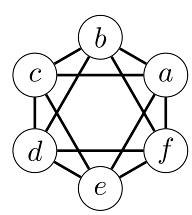
   (c) For which n does Q_n have an Eulerian circuit? Explain.

2. [**4 points**] Find an Eulerian trail in the following graph.



3. [**4 points**] Give an example of a 4-regular planar graph without loops or multiple edges. (Recall that a graph is k-regular if every vertex has degree k.)

4. [**4 points**] Is the following graph planar or nonplanar? If it is planar, give a planar drawing. If not, find a subgraph homeomorphic to K_5 or $K_{3,3}$.



5. **[2 parts, 4 points each]** Let G be a 10-vertex graph with 26 edges and without loops or multiple edges.

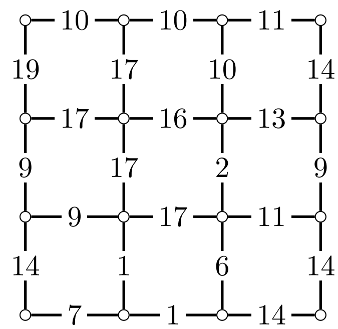
   (a) Show that G is not a planar graph.

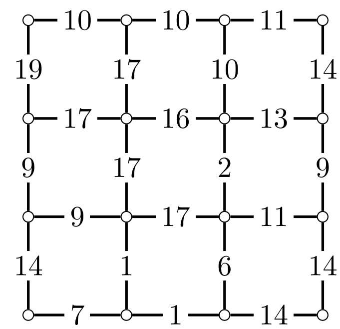
   (b) Show that in every drawing of G in the plane, at least *three* edges of G are involved in edge crossings.

6. **[4 points]** Let G be a connected planar graph with 328 edges without loops and multiple edges. In a planar drawing of G, the boundary of every region contains at least 8 edges. How many vertices must G contain? Give the best lower bound you can.

7. **[4 points]** A connected 50-vertex graph G with 100 edges is drawn in the plane. How many regions are there in the drawing?

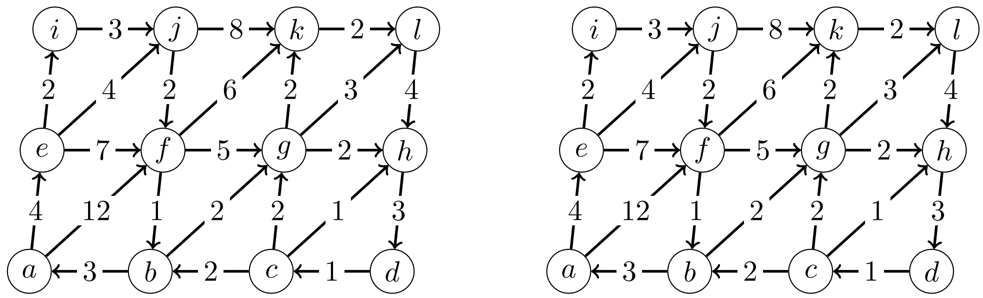
8. **[4 points]** Find a minimum weight spanning tree in the following graph (2 copies).





9. **[4 points]** Let G be a connected graph without loops or multiple edges on at least 3 vertices. Suppose that G has distinct edge weights, and let e be the *second* lightest edge. Prove or disprove: the minimum weight spanning tree of G contains e.

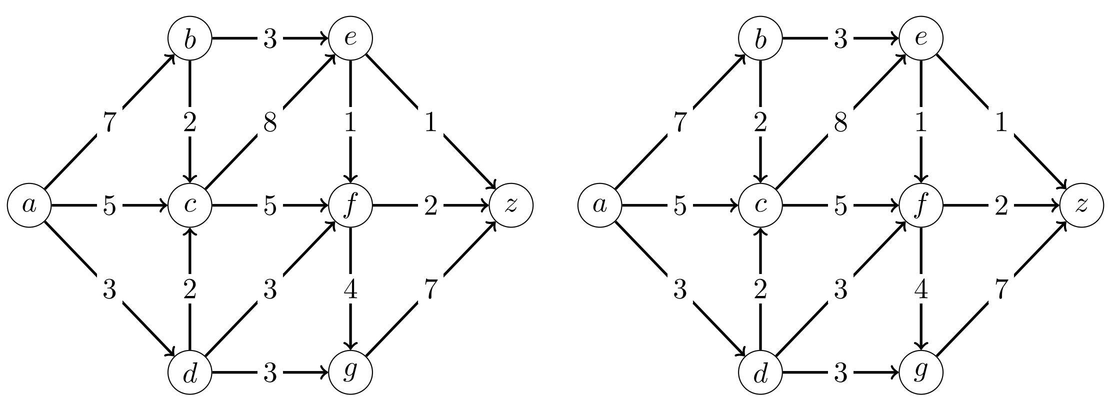
10. **[2 parts, 3 points each]** Consider the following directed graph (2 copies).



(a) Use Dijkstra's algorithm to find the distance between a and all other vertices.

(b) Find a shortest path from a to k.

11. **[2 parts, 3 points each]** Consider the following network N (2 copies).



(a) Find a flow in N with value 10. (Indicate flow values on a copy of the network above; clearly mark which copy contains your answer.)

(b) Find a cut $(P, \overline{P})$ in N of capacity 10. (Indicate the cut by circling a set of vertices in the network above; clearly mark which circle represents the cut.)