Name: _____

Directions: Show all work. No credit for answers without work.

1. [**2 parts, 2 points each**] Consider the group $(\mathbb{Z}_7^*, \odot)$ where $x \odot y = xy \mod 7$.

   (a) Construct the operation table for the group.

   (b) What is the inverse of 3?

2. Let $W = Z_2^3$ and let $E \colon W \to \mathbb{Z}_2^9$ be the encoding function which repeats a message 3 times, so that $E(w) = www$. Let $D \colon \mathbb{Z}_2^9 \to W$ be the corresponding decoding function. The channel flips bits with probability $p = 0.20$.

   (a) [**1 point**] If the received codeword is $r = 100110010$, find the decoded message $D(r)$.

   (b) [**1 point**] In part (a), what can you say about the number of errors that occurred during transmission?

   (c) [**2 points**] If a message w in W is transmitted *without* encoding, what is the probability that w is received without error?

   (d) [**2 points**] What is the probability that a transmission is successful with the encoding scheme?