Math343 Quiz 7 October 12, 2012

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[6 points]** Find $\det\left(\begin{bmatrix} 2 & 1 & 4 & 3 \\ 2 & 1 & 3 & 3 \\ 2 & 1 & 1 & 4 \\ 1 & 3 & 1 & 2 \end{bmatrix}\right)$. *Hint*: this will be easier if you first perform some elementary operations.

$$\begin{vmatrix} 2 & 1 & 4 & 3 \\ 2 & 1 & 3 & 3 \\ 2 & 1 & 1 & 4 \\ 1 & 3 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 4 & 3 \\ 0 & 0 & -1 & 0 \\ 2 & 1 & 1 & 4 \\ 1 & 3 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 4 & 3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -3 & 1 \\ 1 & 3 & 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -5 & 2 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -3 & 1 \\ 1 & 3 & 1 & 2 \end{vmatrix} = (-1)\,\underbrace{(1)(-5)(-1)(1)}_{\text{matrix entries}} = \boxed{-5}$$

~~neg~~ odd permutation

matrix entries

only non-zero permutation

3 inversions, negative sign

2. Let $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : (x-3)^2 + (y-2)^2 \leq 9 \right\}$. That is, S is the disc centered at $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ with radius 3. Let $A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$, and let $T = \{A\mathbf{u} : \mathbf{u} \in S\}$.

(a) **[1 point]** Find the area of S.

$$\text{Area}(S) = \pi r^2 = \pi \cdot 3^2 = \boxed{9\pi}$$

[3 points]
(b) Find the area of T.

$$\text{Area}(T) = \det(A) \cdot \text{Area}(S)$$

$$= \begin{vmatrix} 3 & -2 \\ 1 & 1 \end{vmatrix} \cdot 9\pi = (3+2)9\pi = \boxed{45\pi}$$