Name: Key

Directions: Show all work. No credit for answers without work.

1. **[4 points]** Let A, B, and C be matrices of appropriate sizes. Find an identity that expresses $(ABC)^T$ in terms of A^T, B^T, and C^T.

$$(ABC)^T = ((AB)C)^T = C^T(AB)^T = C^T B^T A^T$$

2. **[6 points]** Prove that if A, B, and C are matrices of appropriate sizes, then $C(A+B) = CA+CB$.

Let A and B be $(p\times n)$-matrices. Let C be an $(m\times p)$-matri

Let $D = A+B$; note that D is an $(p\times n)$-matrix with

$$d_{ij} = a_{ij} + b_{ij}.$$

Let $E = CD$. Note that E is an $(m\times n)$-matrix and

$$e_{ij} = \sum_{k=1}^{p} c_{ik}d_{kj} = \sum_{k=1}^{p} c_{ik}(a_{kj}+b_{kj})$$

$$= \sum_{k=1}^{p} c_{ik}a_{kj} + \sum_{k=1}^{p} c_{ik}b_{kj}.$$

Let $F = CA$ and $G = CB$, and note that F and G are $(m\times n)$-matr with

$$f_{ij} = \sum_{k=1}^{p} c_{ik}a_{kj}, \quad g_{ij} = \sum_{k=1}^{p} c_{ik}b_{kj}.$$

Let $H = F+G$; note that H is an $(m\times n)$-matrix with

$$h_{ij} = f_{ij} + g_{ij} = \sum_{k=1}^{p} c_{ik}a_{kj} + \sum_{k=1}^{p} c_{ik}b_{kj}.$$

Since $e_{ij} = h_{ij}$, we have $E = H$ and $C(A+B) = CA + CB$. $\blacksquare$