Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[3 points]** Given that $L\colon P_2 \to P_1$ is a linear transformation and

$$L(t^2) = 2t + 4 \qquad L(t) = -3t + 6 \qquad L(1) = t + 2,$$

find $L(2t^2 + 3t - 1)$.

$$L(2t^2+3t-1) = 2L(t^2) + 3L(t) - L(1)$$

$$= 2(2t+4) + 3(-3t+6) - (t+2) = \boxed{-6t + 24}$$

2. **[3 points]** Let $L\colon M_{22} \to \mathbb{R}^2$ be the linear transformation given by

$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a-d \\ b-c \end{bmatrix}.$$

Find a basis for $\ker L$.

$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \vec{0}$$

$$\begin{bmatrix} a-d \\ b-c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$a - d = 0$

$b - c = 0$

$c = r_1,\ d = r_2,\ a = r_2,\ b = r_1$

$$\ker L = \left\{ \begin{bmatrix} r_2 & r_1 \\ r_1 & r_2 \end{bmatrix} : r_1, r_2 \in \mathbb{R} \right\}$$

$$= \left\{ r_2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + r_1\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} : r_1, r_2 \in \mathbb{R} \right\}$$

So, basis for ker L:

$$\boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$$

3. [**4 points**] We define a function $L\colon \mathbb{R}^2 \to \mathbb{R}^2$ by

$$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} 3u_2 - u_1 \\ u_1 + u_2 \end{bmatrix}.$$

Is L a linear transformation? Justify your answer.

Soln 1: $\boxed{\text{Yes,}}$ because L is the matrix transformation

$$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where the standard matrix representing L is $\begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$.

Soln 2: We ~~compare~~ check $L\left(a\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + b\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) \stackrel{?}{=} aL\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) + bL\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right)$

LHS: $L\left(a\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + b\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = L\left(\begin{bmatrix} au_1 + bv_1 \\ au_2 + bv_2 \end{bmatrix}\right)$

$$= \begin{bmatrix} +3(au_2 + bv_2) - (au_1 + bv_1) \\ (au_1 + bv_1) + (au_2 + bv_2) \end{bmatrix} = \begin{bmatrix} 3au_2 + 3bv_2 - au_1 - bv_1 \\ au_1 + bv_1 + au_2 + bv_2 \end{bmatrix}$$

RHS: $aL\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) + bL\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = a\begin{bmatrix} 3u_2 - u_1 \\ u_1 + u_2 \end{bmatrix} + b\begin{bmatrix} 3v_2 - v_1 \\ v_1 + v_2 \end{bmatrix}$

$$= \begin{bmatrix} a(3u_2 - u_1) + b(3v_2 - v_1) \\ a(u_1 + u_2) + b(v_1 + v_2) \end{bmatrix}$$

$$= \begin{bmatrix} 3au_2 - au_1 + 3bv_2 - bv_1 \\ au_1 + au_2 + bv_1 + bv_2 \end{bmatrix}$$

Since LHS = RHS, $\boxed{\text{yes}}$, L is a linear transformation.