

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [3 points] Give an example of a 3-dimensional subspace of P , the vector space of all polynomials.

$$\boxed{\text{Span} \{1, t, t^2\}} \quad \text{or} \quad \boxed{P_2} \quad \text{or} \dots$$

many answers possible.

2. Let $A = \begin{bmatrix} 1 & 3 & 1 & 1 & -2 \\ 2 & 6 & 3 & 4 & -3 \\ 3 & 9 & 1 & -1 & -8 \end{bmatrix}$.

- (a) [6 points] Find bases for the row space, the column space, and the null space of A . Clearly indicate which basis is for which space.

rref(A): $\begin{bmatrix} 1 & 3 & 1 & 1 & -2 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & -2 & -4 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & -1 & -3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\uparrow \quad a \quad \uparrow \quad b \quad c$

row space:

$$\begin{bmatrix} 1 \\ 3 \\ 0 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} \quad \text{or} \quad [1 \ 3 \ 0 \ -1 \ -3], [0 \ 0 \ 1 \ 2 \ 1]$$

column space:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Note: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is not correct.

null space:

$$\begin{bmatrix} -3a + b + 3c \\ a \\ -2b - c \\ b \\ c \end{bmatrix} \sim \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

- (b) [1 point] Find the rank of A and the nullity of A .

$$\text{rank}(A) = \dim(\text{row space}) = \dim(\text{col space}) = \boxed{2}$$

$$\text{nullity}(A) = \dim(\text{null space}) = \boxed{3}$$