Practice Test 1 Solns

1. $$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & & \\ -3 & 1 & -6 & & 1 & \\ 0 & 0 & 1 & & & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -2 \\ 0 & 1 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

So inverse is $\begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

2. $$\begin{bmatrix} 1 & 2 & -2 & 4 \\ 2 & 4 & -1 & 17 \\ -1 & -2 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -2 & 4 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -2 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -2 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 10 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3. $$x_1 + 8x_2 - 3x_4 = 6$$

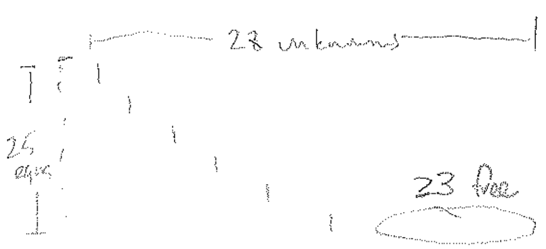
$$x_3 + 4x_4 = 7$$

$x_4 = a, \quad x_3 = 7 - 4a$

$x_2 = b, \quad x_1 = 6 + 3a - 8b.$

For all a and b.

4. Determinants not on test 1. But, answers are: -30, 0, -20.

5. 

Either there will be zero solutions (if the system is inconsistent), or there will be infinitely many, since ≥ 3 unknowns will be free/unconstrained

6. $B = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

a. $\frac{1}{8}r_1 \to r_1$ $\left(E = \begin{bmatrix} \frac{1}{8} & 0 \\ 0 & 1 \end{bmatrix} \right)$

$$EB = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$-3r_1 + r_2 \to r_2$ $\left(D = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \right)$

$$DEB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b. $\underbrace{DE}_{B^{-1}}B = I_2$

so $B^{-1} = DE$

$$= \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{8} & 0 \\ 0 & 1 \end{bmatrix}$$

$B = E^{-1}D^{-1}$

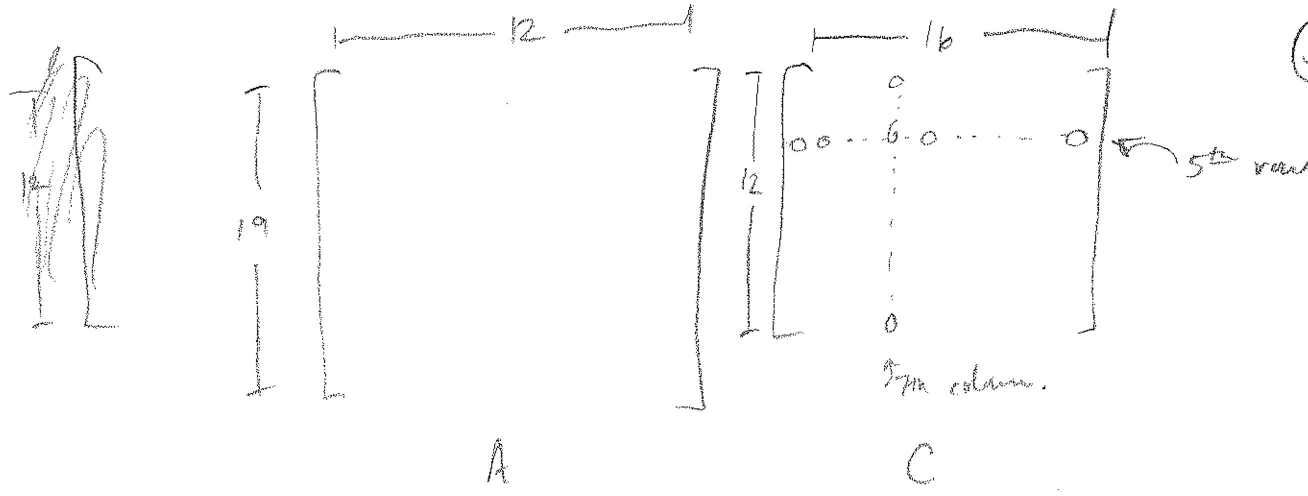
$$= \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

c. When is $\begin{bmatrix} 8 & 4 \\ 5 & c \end{bmatrix}$ invertible?

$$\begin{bmatrix} 8 & 4 \\ 5 & c \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{1}{2} \\ 5 & c \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & c - \frac{5}{2} \end{bmatrix}$$

As long as $c - 5/2 \neq 0$, this matrix ~~[illegible]~~ is invertible and therefore the product of elementary matrices. So, the set $\{c \in \mathbb{R} : c \neq 5/2\}$ is the set of all real numbers for which the matrix is the product of elementary matrices.

7.



We know AC is a (19×16)-matrix whose 7^{th} column is all zeros, so there are <u>at least</u> <u>19 zeros in AC.</u>

(We cannot say anything more about the 5^{th} row.)

8. ~~Ab~~ No. If A is (4×4)-matrix and $Ax = \begin{bmatrix}1\\2\\0\\0\end{bmatrix}$ has a unique solution, then A is invertible. Hence, $Ax = \begin{bmatrix}0\\1\\2\\0\end{bmatrix}$ must have a solution:

$$Ax = \begin{bmatrix}0\\1\\2\\0\end{bmatrix}$$

$$A^{-1}Ax = A^{-1}\begin{bmatrix}0\\1\\2\\0\end{bmatrix}$$

$$x = A^{-1}\begin{bmatrix}0\\1\\2\\0\end{bmatrix}.$$

9. Each point (x,y) gives a linear equation the conic curve must satisfy; for example, including the point $(2,1)$ requires

$$4A + 2B + C + 2D + E + F = 0.$$

Hence, if we have five points, we get a system of 5 linear equations in 6 unknowns ($A, B, C, D, E,$ and F).

Because the system is homogeneous, it is consistent ~~because~~ (the trivial solution $A = B = C = D = E = F = 0$ satisfies all equations). Because there are more unknowns than equations, we get infinitely many non-trivial solutions; these are conic curves containing the required 5 points. $\blacksquare$