

The following lists material that will be covered on Test 3. In addition to this list, review the WileyPLUS homeworks, the quizzes, and class notes. It is possible that a test question will touch on material that I have forgotten to list here. Also, due to time constraints, not all material listed here will appear on Test 3. Material that is listed but does not appear on Test 3 becomes a likely topic for the final exam.

1. 4.2: Inflection Points

- (a) Concavity of functions
- (b) If  $x$  is a point of inflection for  $f$ , then  $f''(x) = 0$ , but not every point  $x$  with  $f''(x) = 0$  is a point of inflection.

2. 4.4: Profit, Revenue, and Cost (Applications of Global Min/Max)

- (a) Global Minimums, global maximums
- (b) Word optimization problems (e.g. a botanical garden with 3 sides of shrubs and 1 side of fencing)
- (c) Find critical points
- (d) Evaluate  $f$  at the critical points and check behavior of  $f$  at the endpoints
- (e) Profit is maximized when Marginal Revenue = Marginal Cost, or at the endpoints of the allowable production range.

3. 4.5: Average Cost

- (a)  $a(q) = \frac{C(q)}{q}$
- (b) Graphical interpretation of average cost:  $a(q)$  is the slope of line from the origin  $(0, 0)$  to the point  $(q, C(q))$  on the cost curve.
- (c) Average cost is minimized when Marginal Cost = Average Cost, or at the endpoints of the allowable production range.

4. 5.1: Distance and Accumulated Change

- (a) Graphical interpretation: given a rate of change  $f'(t)$  (e.g. velocity), the total change in  $f$  over an interval  $[a, b]$  (e.g. the distance traveled from time  $t = a$  to time  $t = b$ ) is given by the area under the graph of  $f(t)$  between  $t = a$  and  $t = b$ .
- (b) We can approximate areas with rectangles.
  - i. The number of rectangles is  $n$ .
  - ii. The width of each rectangle is  $\Delta t$ , and  $\Delta t = \frac{b-a}{n}$ .
  - iii. The height of each rectangle is either taken from the left side of the rectangle (for a Left Hand Sum) or from the right side of the rectangle (for a Right Hand Sum).
  - iv. The area under the curve is approximately the sum of the areas of the rectangles.
  - v. The more rectangles (i.e. the larger  $n$  is and the smaller  $\Delta t$  is), the better the approximation.
  - vi. The Left Hand Sum and the Right Hand Sum are examples of *Riemann sums*.

5. 5.2: The Definite Integral

- (a) The definite integral is the limit of Riemann sums.

- (b) Estimation of definite integrals using Riemann sums (Left Hand Sum and Right Hand Sum).
  - (c) Know when a Riemann sum gives an upper bound or a lower bound on the definite integral. (Note: in some cases, we won't be able to tell.)
6. 5.3: The Definite Integral as Area
- (a) Interpretation of the definite integral as a signed area.
  - (b) Regions below the  $x$ -axis contribute negatively, regions above contribute positively.
7. 5.4: Interpretations of the Definite Integral
- (a) Units of a definite integral.
  - (b) Interpreting between definite integrals, areas of regions, and word problems.
  - (c) See warm-up problems for some examples.
8. 5.5: Fundamental Theorem of Calculus
- (a) Know the Fundamental Theorem of Calculus.
  - (b) FTC tells us that the definite integral of a rate of change equals the total change.
9. 7.1: Finding Antiderivatives Analytically
- (a) Antiderivatives
  - (b) The indefinite integral
  - (c) Basic integration rules
    - i.  $\int k dx = kx + C$
    - ii. If  $n \neq -1$ , then  $\int x^n dx = \frac{x^{n+1}}{n+1}$ .
    - iii. Sum and constant multiple rules:  $\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$  and  $\int cf(x) dx = c \int f(x) dx$  where  $c$  is a constant.
    - iv.  $\int \frac{1}{x} dx = \ln|x| + C$
    - v. If  $k \neq 0$ , then  $\int e^{kx} dx = \frac{1}{k}e^{kx} + C$ .
10. 7.2: Integration by Substitution
- (a) Use substitution to evaluate indefinite integrals
11. 7.3: Using FTC to find Definite Integrals
- (a) Use FTC and antiderivatives to find the value of definite integrals
  - (b) Definite integrals and substitution