Announcements

- HW 8 due Fri
- Quiz 8 out Thurs; due Tues
- Mid-semester feedback today

5.1 Differentiation: given how a quantity (e.g. distance) depends on another (eg. time), how do we find the rate of change (e.g. velocity)?

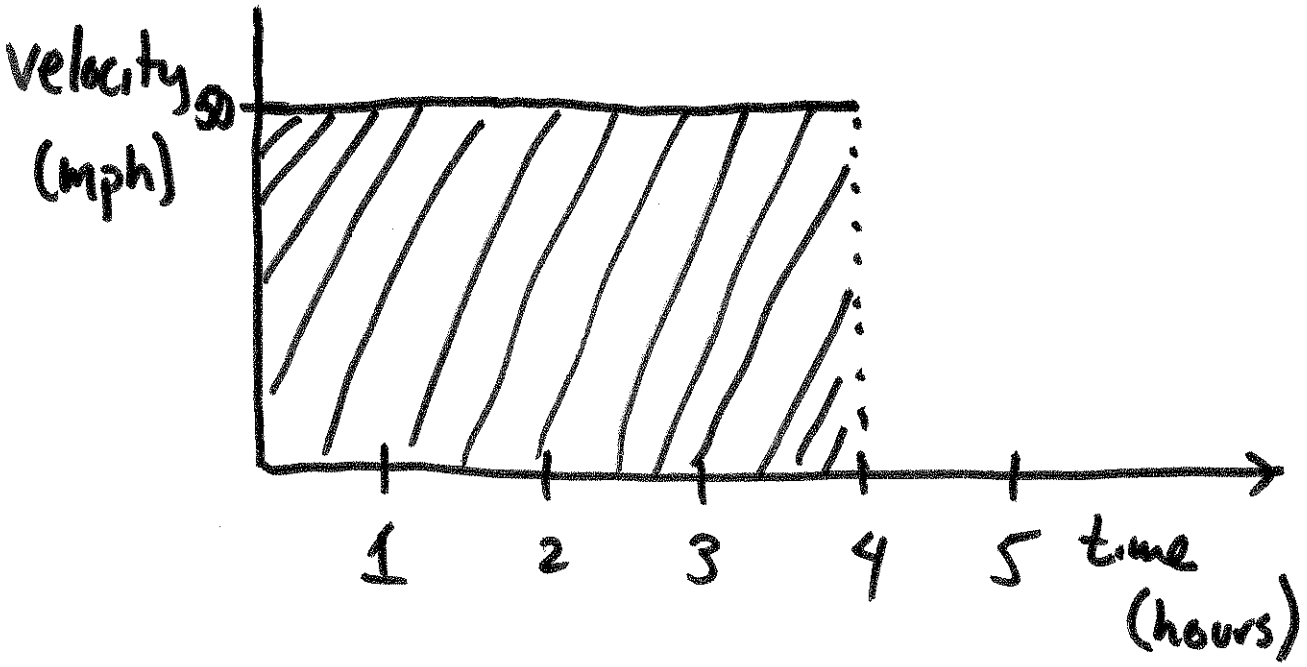
- Integration: given a rate of change (e.g. velocity), how do we recover the original relationship between the two quantities (e.g. distance and time)?

Ex: Suppose a car travels at 50 mph for 4 hours. How far does the car travel?

Soln: Distance = velocity · time

= 50 mph · 4 hrs

= 200 miles



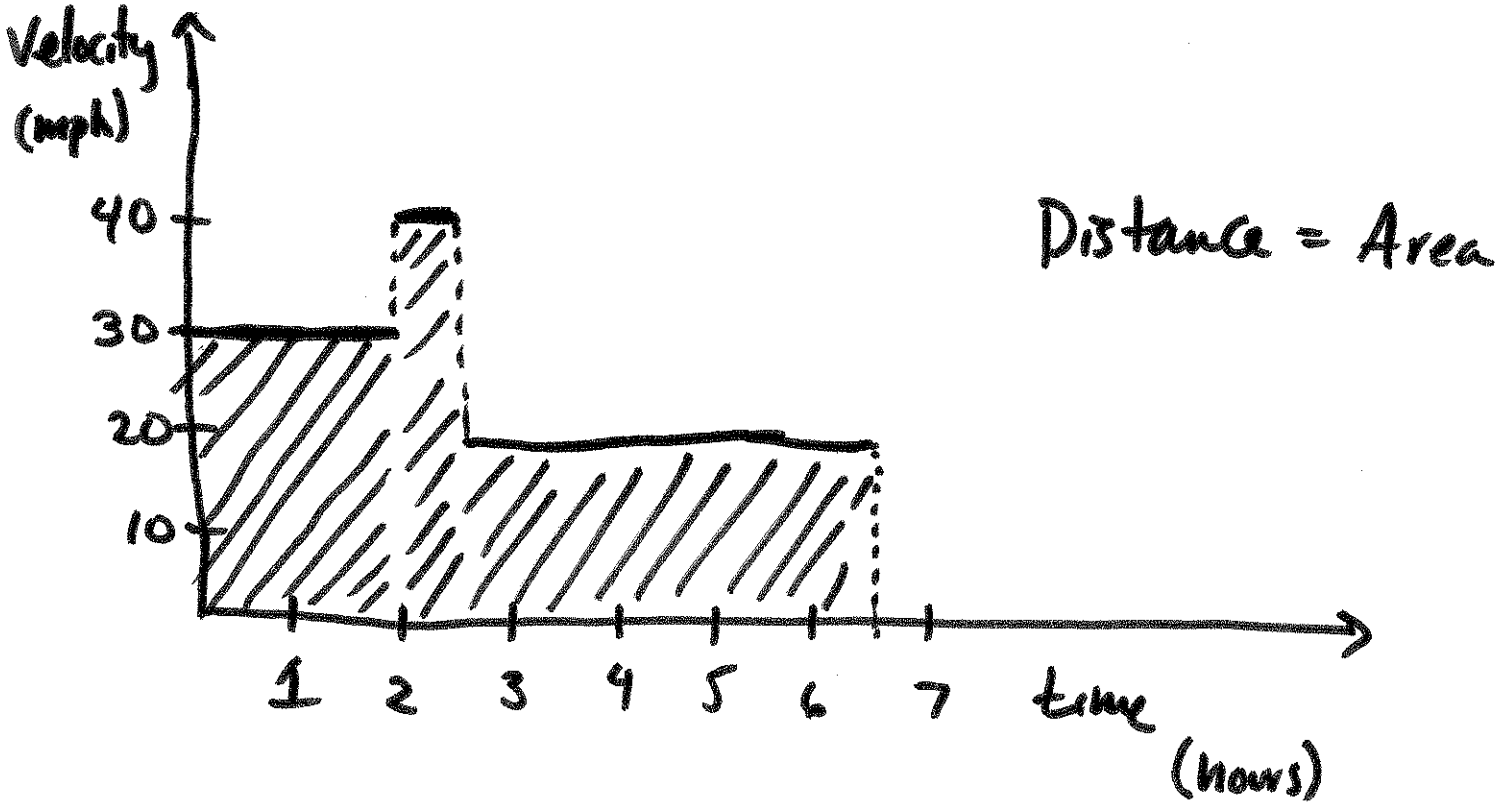
Distance = Area

Ex Suppose a car travels at a speed of 30 mph for 2 hours, then 40 mph for $\frac{1}{2}$ hour, and finally 20 mph for 4 hours. How far does the car travel?

Soln Distance $= 30 \cdot 2 + 40 \cdot \frac{1}{2} + 20 \cdot 4$

$= 60 + 20 + 80$

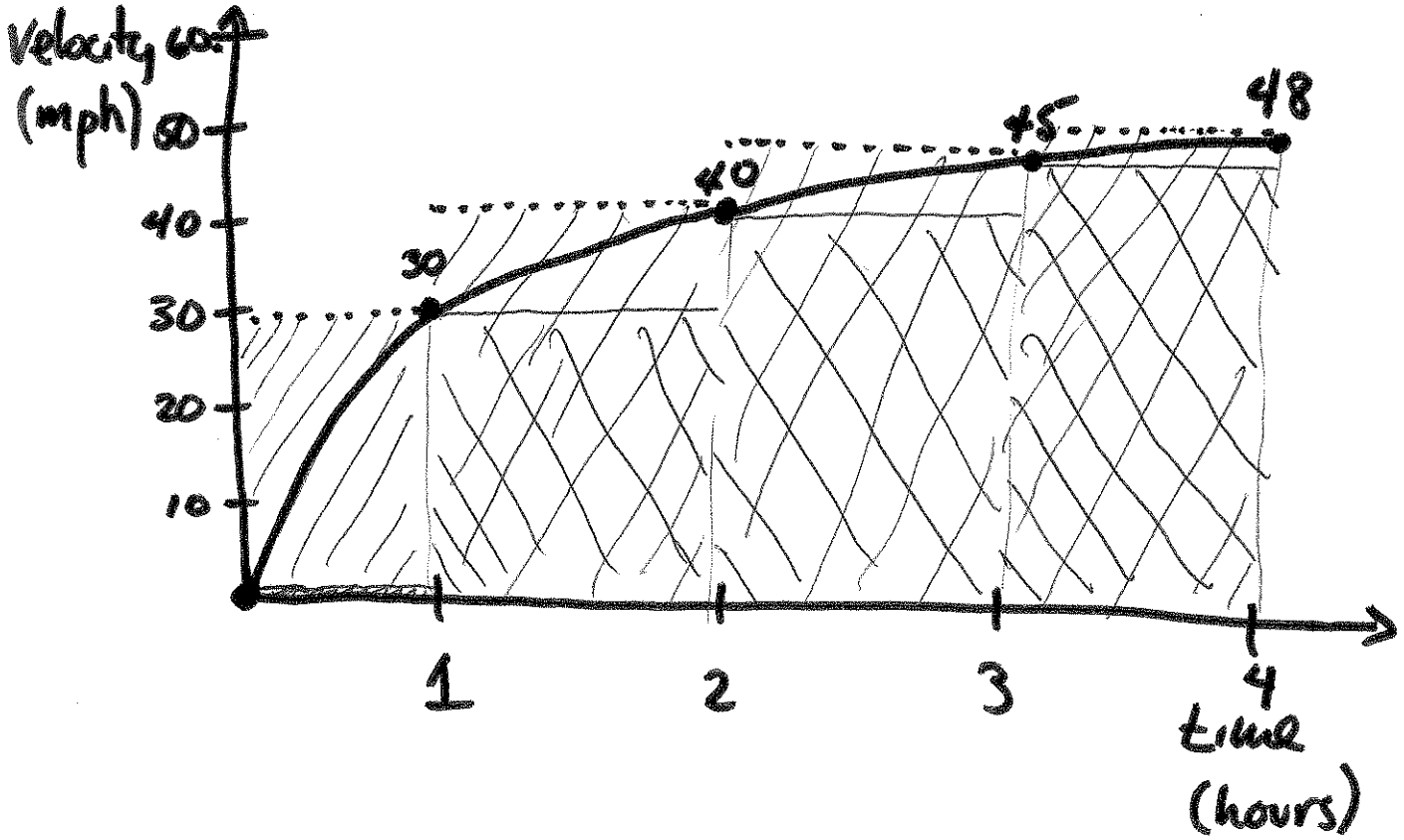
$= 160$ miles



at time t (in hours)

Ex: Suppose a car travels at a speed of $\frac{60t}{1+t}$ mph. How far did the car travel after 4 hours?

Soln



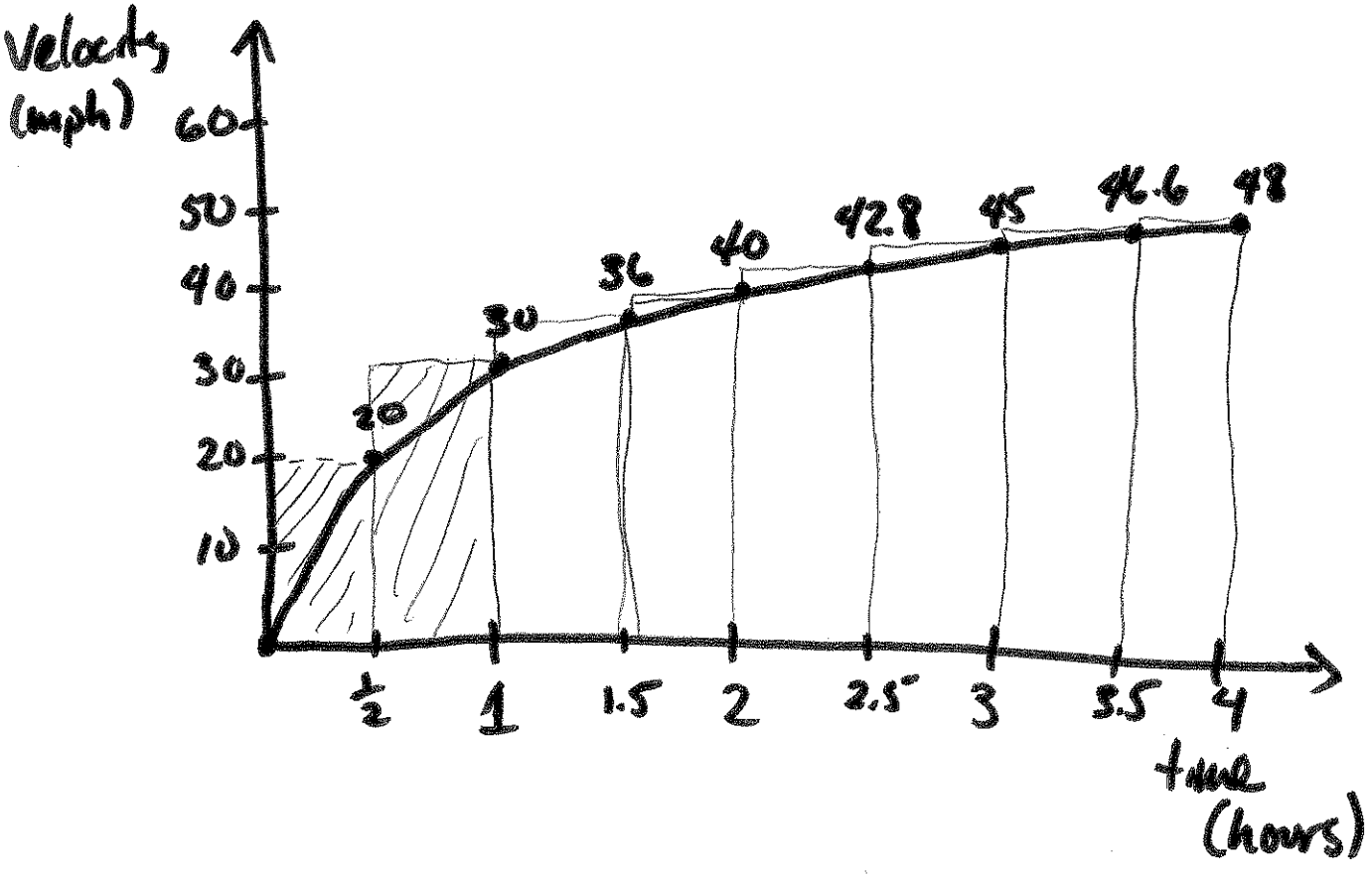
- Distance $\leq 30 \cdot 1 + 40 \cdot 1 + 45 \cdot 1 + 48 \cdot 1$

$= 163$ miles

- Distance $\geq 0 \cdot 1 + 30 \cdot 1 + 40 \cdot 1 + 45 \cdot 1$

≥ 115 miles

- Know car traveled between 115 and 163 miles.
- If we use $\frac{1}{2}$ hr intervals,



- Using 8 rectangles,

$$\text{Distance} \leq \tfrac{1}{2}\cdot 20 + \tfrac{1}{2}\cdot 30 + \cdots$$

$$\leq 154.23 \text{ miles}$$

$$\text{Distance} \geq \tfrac{1}{2}\cdot 0 + \tfrac{1}{2}\cdot 20 + \cdots$$

$$\geq 130.23 \text{ miles.}$$

- Exact distance = 143.43 miles.