Announcements

- HW 5 due 11pm
- Quiz 5 out; due Tuesday: Careful About Units!
- OH Today: 2:20pm − 3:20pm; 4:30pm − 5:00pm

Section 3.1:

Ex: Find the equation of the tangent line at $x=2$ to the graph of the function

$$f(x) = x^2(x-4).$$

Soln:

- $f(x) = x^2(x-4)$
  $= x^3 - 4x^2$
- $f'(x) = \frac{d}{dx}\left[x^3 - 4x^2\right]$
  $= \frac{d}{dx}\left[x^3\right] - \frac{d}{dx}\left[4x^2\right]$
  $= 3x^{3-1} - 4\frac{d}{dx}\left[x^2\right]$
  $= 3x^2 - 4(2x^{2-1})$
  $= 3x^2 - 4(2x) = \boxed{3x^2 - 8x}$

- $y = f(a) + f'(a)(x-a)$ ← Tan. line eqn at $x=a$ to f.

- So, for : tangent line at $x=2$, plug in $a=2$.

$$y = f(2) + f'(2)(x-2)$$

- Recall: $f(x) = x^3 - 4x^2$
- $f(2) = 2^3 - 4(2)^2$

$$= 8 - 4 \cdot 4$$
$$= 8 - 16 = -8$$

$f'(x) = 3x^2 - 8x$

- $f'(2) = 3(2)^2 - 8 \cdot 2$

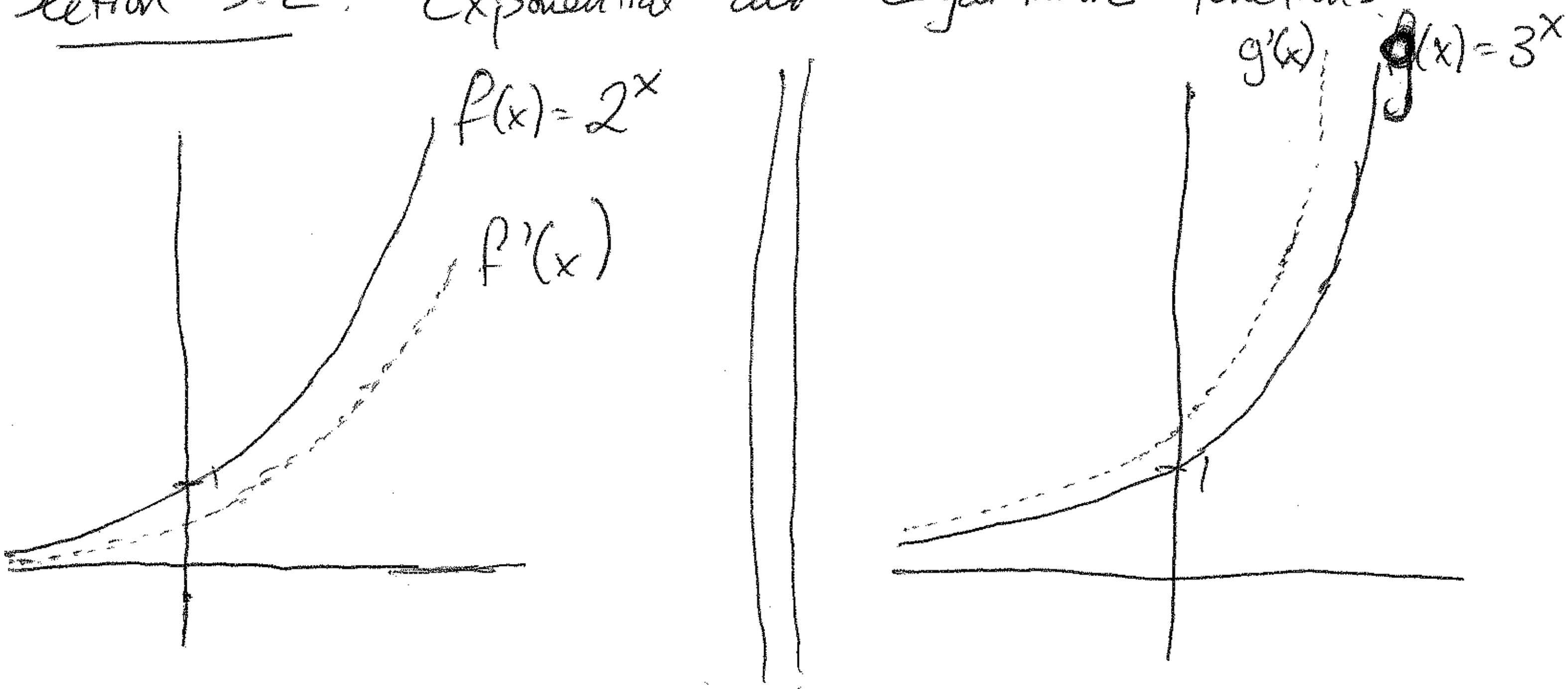
$$= 3 \cdot 4 - 16$$
$$= 12 - 16 = -4$$

- Plug in values into tan line eqn:

$$y = f(2) + f'(2)(x-2)$$
$$y = -8 + (-4)(x-2)$$
$$y = -8 + (-4x + 8)$$
$$= \cancel{-8} + -4x + \cancel{8}$$
$$= -4x$$

- Eqn of tangent line $\boxed{y = -4x}$.

Section 3.2: Exponential and Logarithmic functions



⇒ There should be some number a between 2 and 3 where $\frac{d}{dx}[a^x] = a^x$

⇒ In fact, when $a = 2.71\cdots = e$, this happens.

- $\frac{d}{dx}\left[e^x\right] = e^x$

- $\frac{d}{dx}\left[a^x\right] = (\ln(a))a^x$

- $\frac{d}{dx}\left[e^{kx}\right] = k e^{kx}$

Ex: $\frac{d}{dr}\left[5^r - 2e^{7r} + r^3\right]$

$$= \frac{d}{dr}\left[5^r\right] - \frac{d}{dr}\left[2e^{7r}\right] + \frac{d}{dr}\left[r^3\right]$$

$$= (\ln(5))\cdot 5^r - 2\frac{d}{dr}\left[e^{7r}\right] + 3r^2$$

$$= (\ln(5))\cdot 5^r - 2\left(7e^{7r}\right) + 3r^2$$

$$= \boxed{\ln(5)\cdot 5^r - 14e^{7r} + 3r^2}$$

Ex: $\frac{d}{dx}\left[\frac{e^{5x}+2}{e^x}\right] = \frac{d}{dx}\left[e^{-x}(e^{5x}+2)\right]$

$$= \frac{d}{dx}\left[e^{-x}\cdot e^{5x} + e^{-x}\cdot 2\right]$$

$$= \frac{d}{dx}\left[e^{-x+5x} + 2e^{-x}\right]$$

$$= \frac{d}{dx}\left[e^{4x} + 2e^{-x}\right]$$

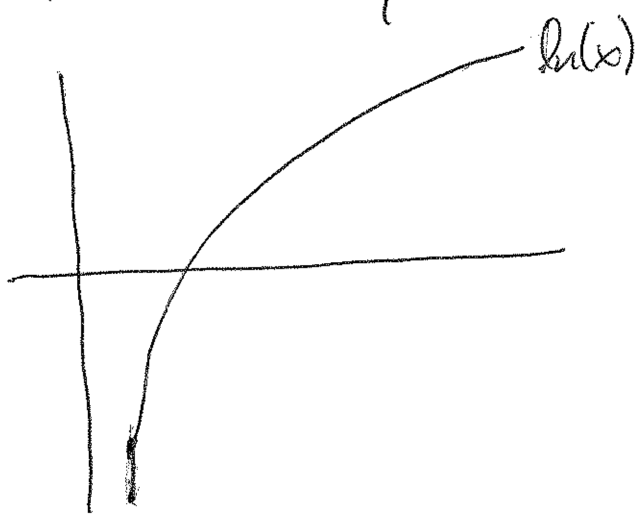
$$= \frac{d}{dx}\left[e^{4x}\right] + 2\frac{d}{dx}\left[e^{-x}\right]$$

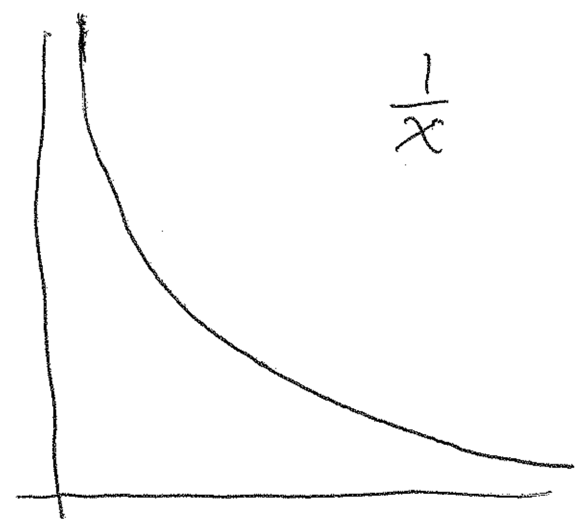
$$\boxed{\frac{d}{dx}\left[e^{kx}\right] = ke^{kx}} \qquad = 4e^{4x} + 2(-1 \cdot e^{-x})$$

$$= \boxed{4e^{4x} - 2e^{-x}}$$

$$= \boxed{4e^{4x} - \frac{2}{e^{x}}}$$

= Derivatives of $\ln(x)$.





- $$\boxed{\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}}$$

- Ex: $\frac{d}{dz}\left[\ln(z) + \frac{1}{z}\right] = \frac{d}{dz}\left[\ln(z)\right] + \frac{d}{dz}\left[\frac{1}{z}\right]$

$$= \frac{1}{z} + \frac{d}{dz}\left[z^{-1}\right]$$

$$= \frac{1}{z} + (-1 \cdot z^{-1-1})$$

$$= \boxed{\frac{1}{z} - z^{-2}}$$

$$= \boxed{\frac{1}{z} - \frac{1}{z^2}}$$

$$\boxed{\ln(A \cdot B) = \ln(A) + \ln(B)}$$

$$\frac{d}{dx}\left[\ln(5x)\right] = \frac{d}{dx}\left[\ln(5) + \ln(x)\right]$$

$$= \frac{d}{dx}\left[\ln(5)\right] + \frac{d}{dx}\left[\ln(x)\right]$$

$$= \frac{d}{dx}\left[1.\overset{6}{?}\overset{1}{?}?\right] + \frac{d}{dx}\left[\ln(x)\right]$$

$$= 0 + \frac{1}{x}$$

$$= \boxed{\frac{1}{x}}$$